



**Solution of non– integer coefficients linear system  
by using mathematical software program when  
the numbers of equations greater than numbe  
of variables**

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**ABSTRACT**

*The system under consecration the number of the equations greater than number of variables and after solve by software programming finded Maxima and SimSolve failed to reach solutions for the system, MATLAB, Excel and Lindo have produced optimal solutions for the system with a nuance between them in the error for the benefit of MATLAB of small error.*

**Keywords:** *linear systems, MATLAB, Excel, Lindo, SimSolv, optimization software*



## Introduction

case study contained a linear system has no single solution, but rather a set of solutions- no simultaneous solution - and the system solution can exist only through the optimization. Maxima and SimSolve failed to reach solutions for the system. Excel , Lindo and MATLAB have produced optimal solutions for the system with a nuance between them in the error for the benefit of MATLAB and Lindo. In the light of these findings, the study recommends the need for introducing of such software within mathematics curriculum and train both teachers and students to use them in order to reduce the time and effort required for systems solutions and take advantage of the ICT to make the teaching and learning process more easily, so as to increase using mathematical models in daily life.

### 1. Case Study:

In this case the number of the equations  $>$  the number of the unknowns Consider the following linear system:

$$6.2 X_1 - 0.3 X_2 + 1.9 X_4 = -22.375$$

$$2.5 X_1 + 3.4 X_4 = -8.75$$

$$0.8 X_1 + 2.1 X_2 - 0.5 X_3 = 2.325$$

$$-0.4 X_2 - 7.4 X_3 + 0.1 X_4 = 5.02$$

$$1.5 X_1 - 8.2 X_2 + 3.1 X_3 = -26.18$$

$$6.2 X_2 + 0.1 X_3 - 8.7 X_4 = 13.87$$

With the solution:

| Variable | Value |
|----------|-------|
|----------|-------|

|       |          |
|-------|----------|
| $X_1$ | $= -3.5$ |
|-------|----------|

|       |          |
|-------|----------|
| $X_2$ | $= 2.25$ |
|-------|----------|

|       |          |
|-------|----------|
| $X_3$ | $= -0.8$ |
|-------|----------|

|       |       |
|-------|-------|
| $X_4$ | $= 0$ |
|-------|-------|

Now substitute the solution in  $1.15 X_1 + 1.8 X_2$  this produce 0.025. Adding this equation ( $1.15 X_1 + 1.8 X_2 = 0.025$ ) to the previous system does not generate a new system, but by changing the constant of this equation from 0.025 to 0.0, a new complicated system will be generated as:

$$6.2 X_1 - 0.3 X_2 + 1.9 X_4 = -22.375$$

$$2.5 X_1 + 3.4 X_4 = -8.75$$

$$0.8 X_1 + 2.1 X_2 - 0.5 X_3 = 2.325$$



$$-0.4 X_2 - 7.4 X_3 + 0.1 X_4 = 5.02$$

$$1.5 X_1 - 8.2 X_2 + 3.1 X_3 = -26.18$$

$$6.2 X_2 + 0.1 X_3 - 8.7 X_4 = 13.87$$

$$1.15 X_1 + 1.8 X_2 = 0$$

## 2. Solution of Case Study by using Excel Solver:

Enter the system as seen in Figure (1)

| A                  | B       | C |
|--------------------|---------|---|
| 0.0000000000000000 | -22.375 |   |
| 0.0000000000000000 | -8.75   |   |
| 0.0000000000000000 | 2.325   |   |
| 0.0000000000000000 | 5.02    |   |
| 0.0000000000000000 | -26.18  |   |
| 0.0000000000000000 | 13.87   |   |
| 0                  | 0       |   |

Figure (1): Case 3 Excel input

Then by solving process Figure (2) will appear:

| C        |
|----------|
| -3.46472 |
| 2.213569 |
| -0.79838 |
| -0.02594 |

Figure (2): Excel Answers

The following Table (1) shows the answers of case study by Excel Solver.

| Table (1): Case Answers of Excel |           |
|----------------------------------|-----------|
| Variable                         | Value     |
| X1                               | - 3.46472 |
| X2                               | 2.213569  |
| X3                               | - 0.79838 |
| X4                               | - 0.02594 |

Table (2) shows the verifying of the solutions by using Excel.



| Table (2): verification of Excel solution |          |                     |                    |
|-------------------------------------------|----------|---------------------|--------------------|
| Equation                                  | Constant | Substitution Value  | The error          |
| Equation 1                                | -22.375  | -22.194610846644400 | -0.180389153355645 |
| Equation 2                                | -8.75    | -8.750001000000000  | 0.000000999999999  |
| Equation 3                                | 2.325    | 2.275912591482200   | 0.049087408517800  |
| Equation 4                                | 5.02     | 5.020000000848900   | -0.000000000848898 |
| Equation 5                                | -26.18   | -25.823327485548900 | -0.356672514451102 |
| Equation 6                                | 13.87    | 13.870000070485100  | -0.000000070485136 |
| Equation 7                                | 0        | 1.32912E-08         | -0.000000013291211 |

### 3. Solution of Case Study by using LINDO:

Enter the system as seen in Figure (3)

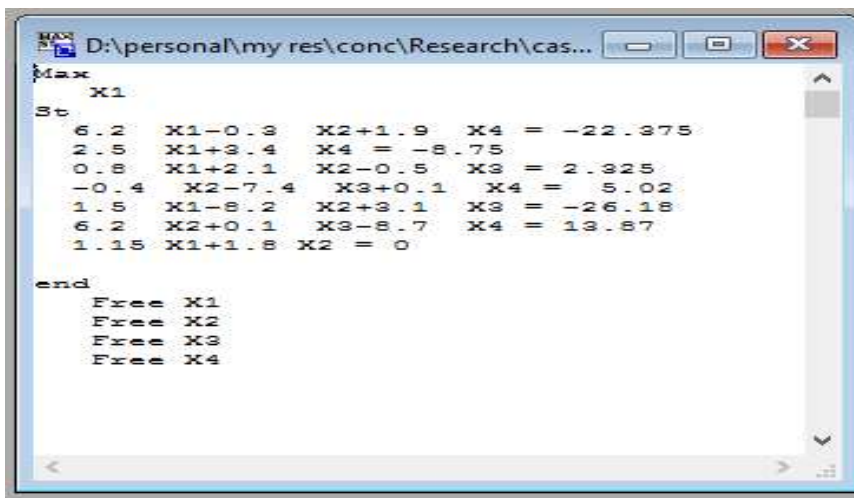
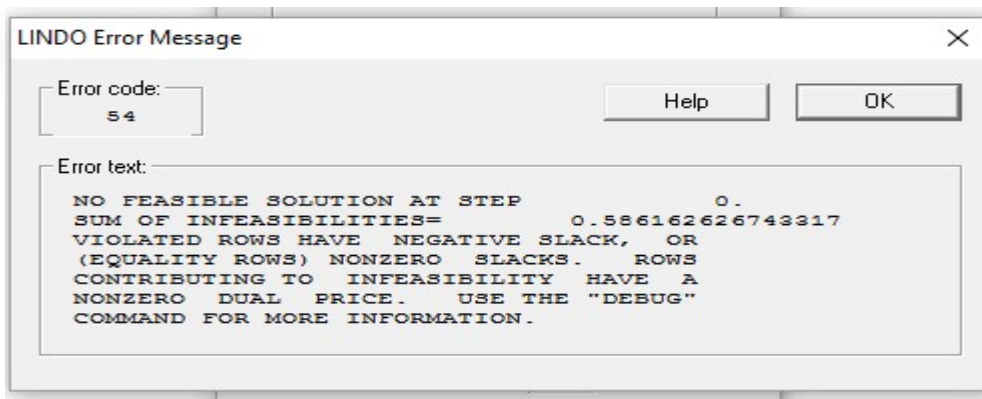
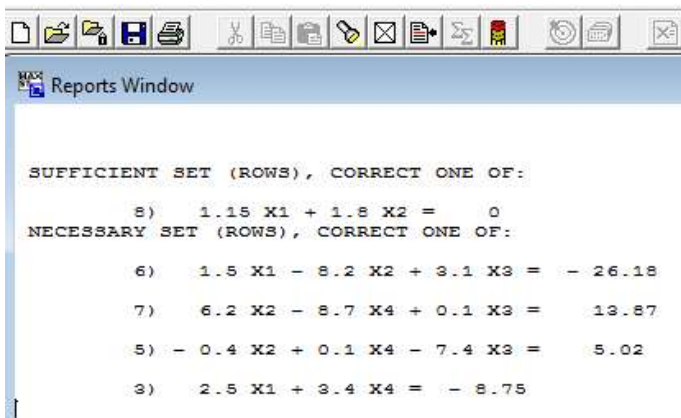


Figure (3): Lindo input

Select solve command an error message will appear, Figure (4)

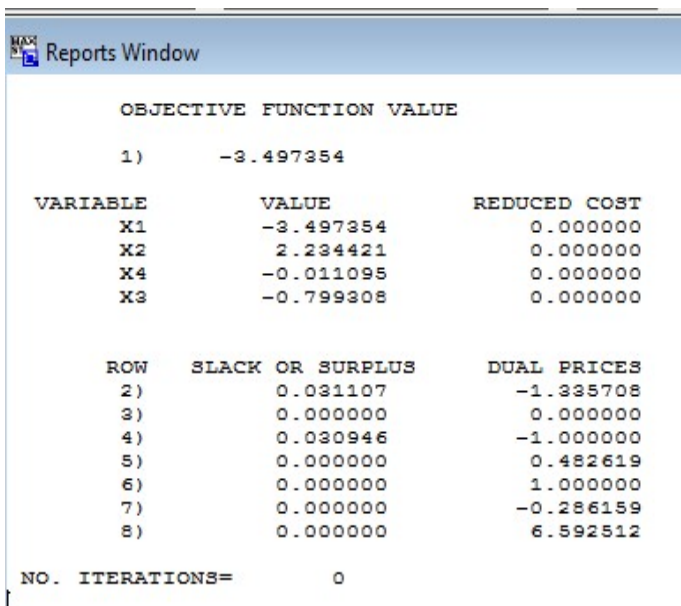


By choosing DEBUG command from solve menu, the report window appear as in Figure (5).



**Figure (5): DEBUG Command**

Then from Report menu choose Solution command, the answer shows as Figure (6)



**Figure (6): Lindo Answers**

The following Table (3) shows the answers of case study 3 by LINDO.



| <b>Table (3): Answers of Lindo</b> |                  |
|------------------------------------|------------------|
| <b>Variable</b>                    | <b>Value</b>     |
| <b>X1</b>                          | <b>-3.497354</b> |
| <b>X2</b>                          | <b>2.234421</b>  |
| <b>X3</b>                          | <b>-0.799308</b> |
| <b>X4</b>                          | <b>-0.011095</b> |

Table (4) shows the verifying of the solutions of Case using Lindo.

| <b>Table (4): Case verification of Lindo solution</b> |                 |                            |                           |
|-------------------------------------------------------|-----------------|----------------------------|---------------------------|
| <b>Equation</b>                                       | <b>Constant</b> | <b>Substitution Value</b>  | <b>The error</b>          |
| <b>Equation 1</b>                                     | <b>-22.375</b>  | <b>-22.375001600000000</b> | <b>0.000001600000001</b>  |
| <b>Equation 2</b>                                     | <b>-8.75</b>    | <b>-8.781108000000000</b>  | <b>0.031108000000000</b>  |
| <b>Equation 3</b>                                     | <b>2.325</b>    | <b>2.294054900000000</b>   | <b>0.030945099999999</b>  |
| <b>Equation 4</b>                                     | <b>5.02</b>     | <b>5.020001300000000</b>   | <b>-0.000001300000001</b> |
| <b>Equation 5</b>                                     | <b>-26.18</b>   | <b>-26.046138000000000</b> | <b>-0.133861999999997</b> |
| <b>Equation 6</b>                                     | <b>13.87</b>    | <b>13.870005900000000</b>  | <b>-0.000005900000003</b> |
| <b>Equation 7</b>                                     | <b>0</b>        | <b>-22.375001600000000</b> | <b>-0.000000700000001</b> |

#### 4. Solution of Case Study using Maxima:

Enter the system as in Figure (7)



Solve linear system

Equation 1:  $2 \cdot X_1 - 0.3 \cdot X_2 + 1.9 \cdot X_4 = -22.375$

Equation 2:  $2.5 \cdot X_1 + 3.4 \cdot X_4 = -8.75$

Equation 3:  $0.8 \cdot X_1 + 2.1 \cdot X_2 - 0.5 \cdot X_3 = 2.325$

Equation 4:  $0.4 \cdot X_2 - 7.4 \cdot X_3 + 0.1 \cdot X_4 = 5.02$

Equation 5:  $5 \cdot X_1 - 8.2 \cdot X_2 + 3.1 \cdot X_3 = -26.18$

Equation 6:  $5.2 \cdot X_2 + 0.1 \cdot X_3 - 8.7 \cdot X_4 = 13.87$

Equation 7:  $1.15 \cdot X_1 + 1.8 \cdot X_2 = 0$

Variables:  $X_1, X_2, X_3, X_4$

OK Cancel

**Figure (7): Maxima input**

Then the output appears, see Figure (8)

```

wolMaxima 16.042 [unsaved]
File Edit View Cell Maxima Equations Algebra Calculus Simplify Plot Numeric Help

[ (%i1)  linsolve([ 6.2*X1-0.3*X2+1.9*X4 = -22.375, 2.5*X1+3.4*X4 = -8.75, 0.8*X1+2.1*X2-0.5*X3 = 2.325, -0.4*X2-7.4*X3+0.1*X4 = 5.02, 5*X1-8.2*X2+3.1*X3 = -26.18, 5.2*X2+0.1*X3-8.7*X4 = 13.87, 1.15*X1+1.8*X2 = 0])
rat: replaced 22.375 by 179/8 = 22.375
rat: replaced 6.2 by 31/5 = 6.2
rat: replaced -0.3 by -3/10 = -0.3
rat: replaced 1.9 by 19/10 = 1.9
rat: replaced 8.75 by 35/4 = 8.75
rat: replaced 2.5 by 5/2 = 2.5
rat: replaced 3.4 by 17/5 = 3.4
rat: replaced -2.325 by -93/40 = -2.325
rat: replaced 0.8 by 4/5 = 0.8
rat: replaced 2.1 by 21/10 = 2.1
rat: replaced -0.5 by -1/2 = -0.5
rat: replaced -5.02 by -251/50 = -5.02
rat: replaced -0.4 by -2/5 = -0.4
rat: replaced -7.4 by -37/5 = -7.4
rat: replaced 0.1 by 1/10 = 0.1
rat: replaced 26.18 by 1309/50 = 26.18
rat: replaced 1.5 by 3/2 = 1.5
rat: replaced -8.2 by -41/5 = -8.2
rat: replaced 3.1 by 31/10 = 3.1
rat: replaced -13.87 by -1387/100 = -13.87
rat: replaced 6.2 by 31/5 = 6.2
rat: replaced 0.1 by 1/10 = 0.1
rat: replaced -8.7 by -87/10 = -8.7
rat: replaced 1.15 by 23/20 = 1.15
rat: replaced 1.8 by 9/5 = 1.8
[ (%o1)  [] ]

```

**Figure (8): Maxima Output**

No solution for Case study given by Maxima

## 5. Solution of Case Study by using SimSolve:

To enter the system in SimSolve choose the number of the variable is 7 because SimSolve built a square matrix by assuming that the number of the variables = number of equations, so there are 3 variables not exist really. By put them as zero values. But this let the matrix has a large number of zero elements as shown in Figure (9).



Number of variables

| A    | B    | C    | D    | E | F | G | RHS     |
|------|------|------|------|---|---|---|---------|
| 6.2  | -0.3 | 0    | 1.9  | 0 | 0 | 0 | -22.375 |
| 2.5  | 0    | 0    | 3.4  | 0 | 0 | 0 | -8.75   |
| 0.8  | 2.1  | -0.5 | 0    | 0 | 0 | 0 | 2.325   |
| 0    | -0.4 | 0    | 0.1  | 0 | 0 | 0 | 5.02    |
| 1.5  | -8.2 | 3.1  | 0    | 0 | 0 | 0 | -26.18  |
| 0    | 0    | 0.1  | -8.7 | 0 | 0 | 0 | 13.87   |
| 1.15 | 1.8  | 0    | 0    | 0 | 0 | 0 | 0       |

Figure (9): SimSolve input

Then appears as shown in Figure (10)

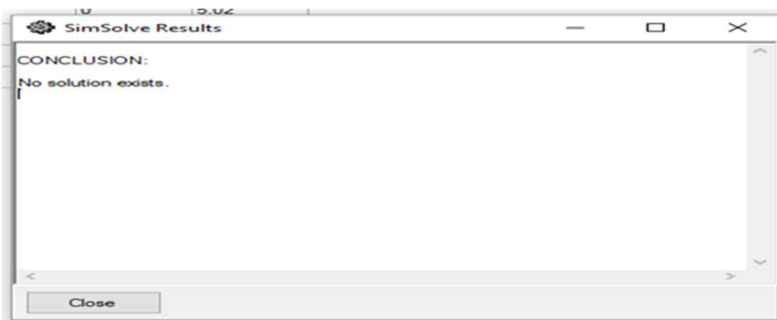


Figure (10): SimSolve Output

No solution exists.

## 6. Conclusion

The number of equations is larger than the number of variables. The system has no simultaneous solution.

|           | Error        |           |           |             |             |
|-----------|--------------|-----------|-----------|-------------|-------------|
|           | Excel Solver | LINDO6.1  | MATLAB    | Maxima      | SimSolve    |
| Equation1 | -0.180389    | 0.000001  | -0.31649  | No Solution | No Solution |
| Equation2 | 0            | 0.031108  | 0.513360  |             |             |
| Equation3 | 0.049087     | 0.030945  | 0.149830  |             |             |
| Equation4 | 0            | -0.000001 | -7.117430 |             |             |
| Equation5 | -0.35667     | -0.133861 | 0.017750  |             |             |
| Equation6 | 0            | -0.000006 | 0.198860  |             |             |
| Equation7 | 0            | 0         | 0.460590  |             |             |
| Avg       | 0.0837351    | 0.027988  |           |             |             |



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