Using Optimization Software for Solving Linear Systems with integer coefficients

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ABSTRACT

This study examines the use of optimization application software to solve linear systems because there are some complex linear systems that do not have a single solution, but an optimal solution among a set of solutions. The study was based on a linear system as a case study, which was solved using Excel and SimSolve as software designed using the traditional methods to solve the linear system under investigation. The same system was also solved again using the optimization software; Excel Solver and Lindo 6.1, where there was no difference in the correct solution in both ways. Then, the linear system, which used as a case study, was modified to become more complex, as the number of equations became greater than the number of variables. When the new system was resolved using the mentioned software, the results were in favor of the optimization software Solver and Lindo 6.1; they succeeded in providing solutions with a margin of error, while, Excel and SimSolve software, designed with traditional algorithms, failed to achieve a result. Based on these results, the study recommends introducing optimization software within the mathematics curriculum in the advanced stages of the educational system and training both the teachers as well as their students to use such software in order to reduce the time and effort required for solving traditional systems, and to obtain best results when solving these systems. It is recommended making use of information and communication technologies that made learning mathematics more easily and increase reliability on mathematical models in solving life problems

Keywords: *linear systems, optimization software, information technology, and life problems.*

Introduction

In this paper we will try to take advantage of the great development in the field of software to employ them in the direction of building mathematical models and resolution.

The aims look for optimization programs to solve systems of linear equations and so these methods requiring less mathematical skills and effort mentally contributes to less than reliable in various applications for non-specialists in mathematics. And then compare this software to find the difference between them and the errors if it is existing. The study helps to shorten the time in the solution of linear systems using some ready-made software with less effort and small errors. Also, through the study note that can non-professionals in the field of mathematics to deal with linear systems.

1. Case Study:

In this case the coefficients of the variables are integer numbers and the number of the equations = number of variables. There are 10 equations of 10 variables x1, x2, x3, . . . , x10

3X1+X2+2X3+4X4+2X5+8X6+4X7-X8-X10 = 7

-X1+3X2+X3-2X4-X5+4X6+5X7-2X8-3X9+7X10 = -28

5X1-X2+3X4+9X5-2X6-X7-4X8+5X9+X10 = 49

2X1+2X2-X3+4X4+8X5-12X6+3X7+2X8-X9-9X10 = 1

X1+11X2-5X3-2X4-X5+8X7+X8-2X9+4X10 = -36

12X1+5X2+2X3+7X5+20X6-9X7-9X8+6X9 =32

8X1-2X2+6X3-5X4+6X5+10X6-13X7-4X9+10X10 = 18

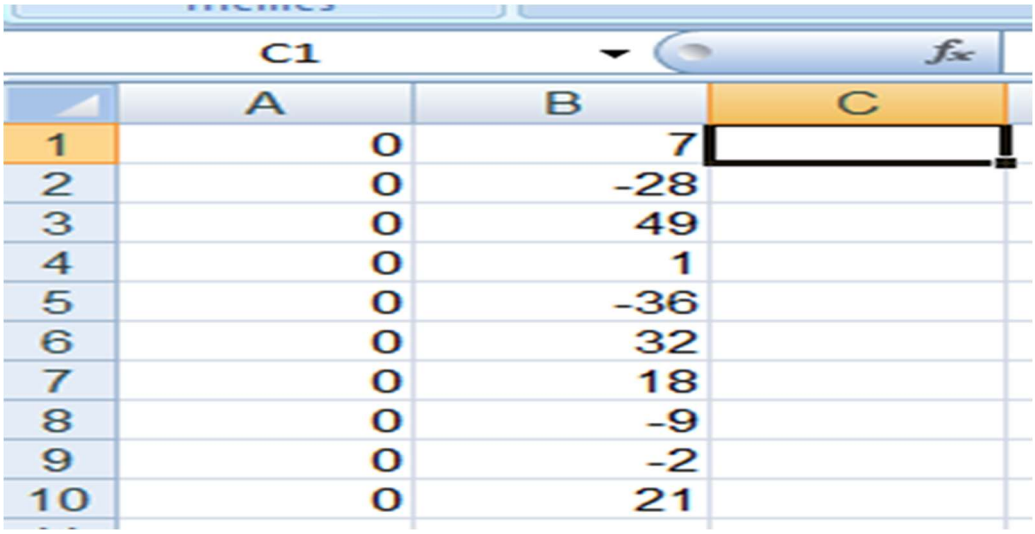
17X2+2X3-X4+3X5+6X6+5X7+2X8-6X9+4X10 = -9

7X1-10X4+2X5-3X7+X8+X9+X10 = -2

13X1+8X2+2X3-7X4+X5-6X6+2X7+3X8+2X9+X10 = 21

2. Solution of case study 1 using Excel Solver:

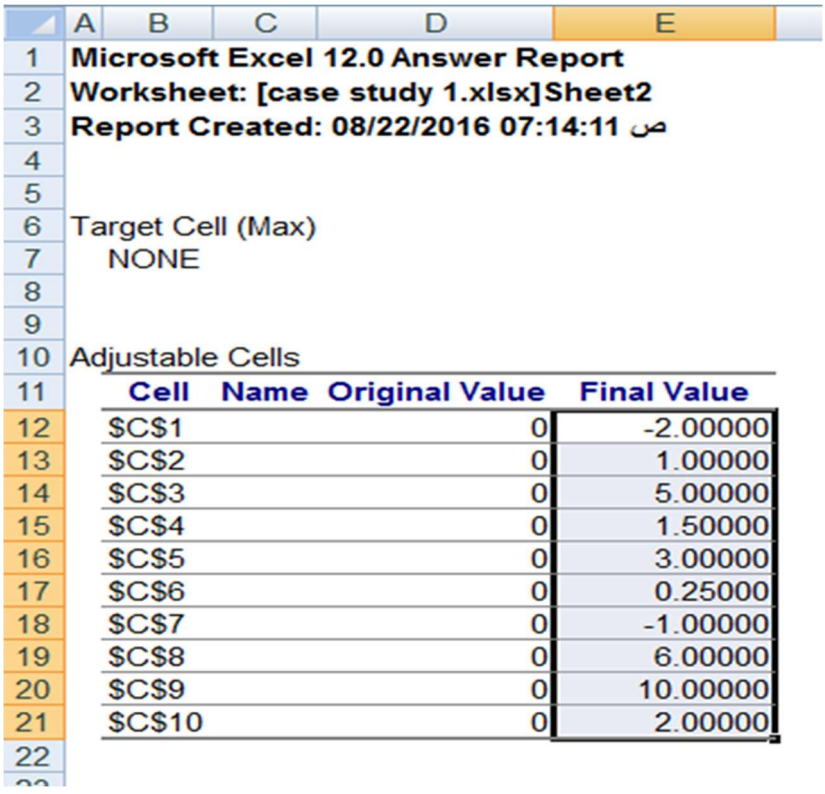
Enter the data (linear system), see Figure (1)



	A	B	C
1	0	7	
2	0	-28	
3	0	49	
4	0	1	
5	0	-36	
6	0	32	
7	0	18	
8	0	-9	
9	0	-2	
10	0	21	

Figure (1): Case Excel input

The solution report is shown in Figure (2)



Microsoft Excel 12.0 Answer Report
Worksheet: [case study 1.xlsx]Sheet2
Report Created: 08/22/2016 07:14:11 ص

Target Cell (Max)
NONE

Adjustable Cells

Cell	Name	Original Value	Final Value
C1		0	-2.00000
C2		0	1.00000
C3		0	5.00000
C4		0	1.50000
C5		0	3.00000
C6		0	0.25000
C7		0	-1.00000
C8		0	6.00000
C9		0	10.00000
C10		0	2.00000

Figure (2): Excel Answers

The following table Table (1) shows the answers by Excel Solver

Table (1): Excel Case 1 Answers										
Variable	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10

Value	−2	1	5	1.5	3	0.25	−1	6	10	2

Table (2) shows the substitutions of the above values of the variables in the first case study to find the errors of the solutions if it exists. The error represents the deference between constant and the final value produced from substitution

Table (2): verification of Excel solution

Equation	Constant	Substitution Value	The error
Equation 1	7	7	0
Equation 2	−28	−28	0
Equation 3	49	49	0
Equation 4	1	1	0
Equation 5	−36	−36	0
Equation 6	32	32	0
Equation 7	18	18	0
Equation 8	=9	=9	0
Equation 9	−2	−2	0
Equation 10	21	21	0

3. Solution of Case Study 1 using LINDO:

Input the system in Lindo like in Figure (3)

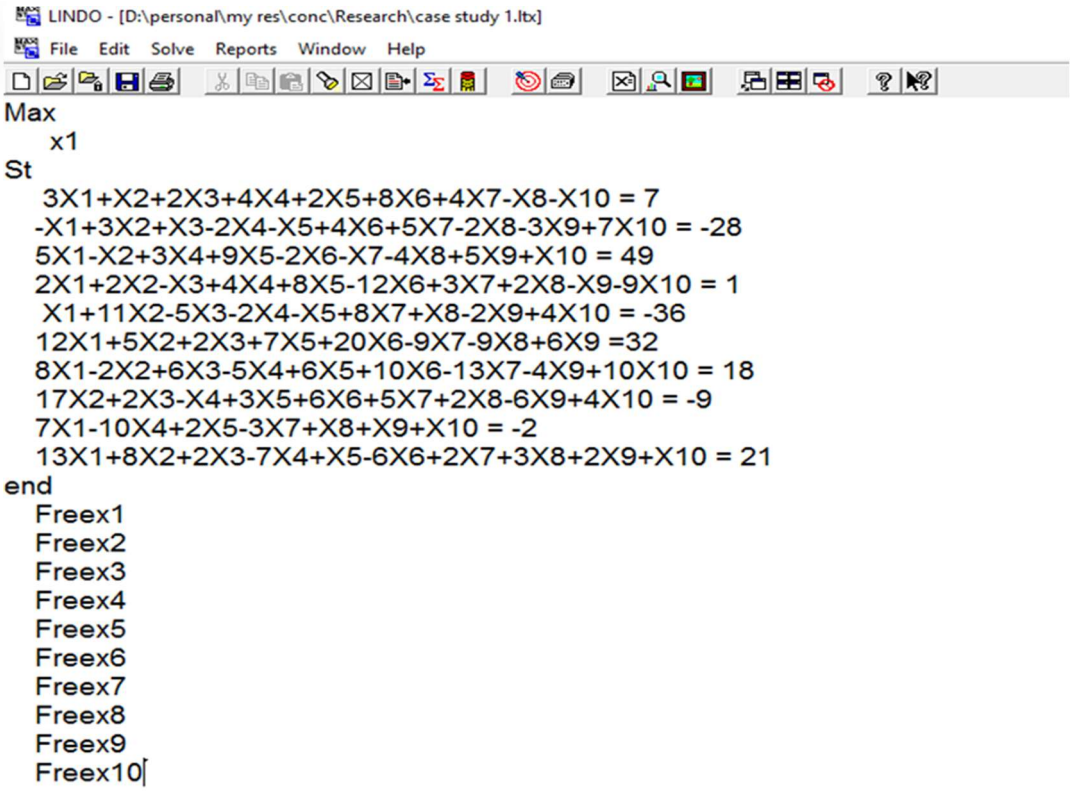


Figure (3): Lindo input

Then the solution seen in Figure (4).

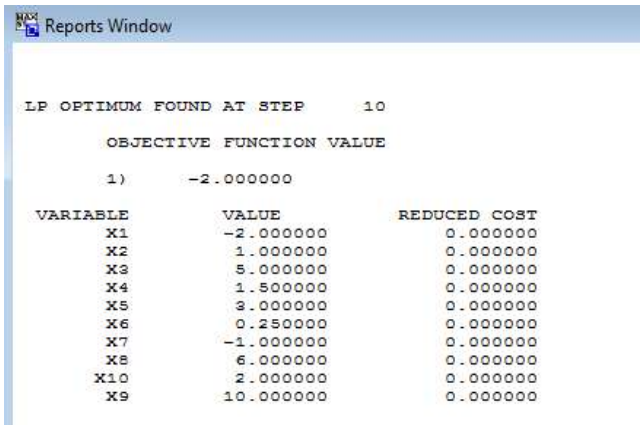


Figure (4): Case 1 Lindo Answer

The following Table (3) shows the answers by LINDO.

Table (3): Lindo Case 1 Answers

Table (3): Lindo Case 1 Answers										
Variable	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10

Value	−2	1	5	1.5	3	0.25	−1	6	10	2

Table (4) shows the substitutions of the above values of the variables in the first case study to find the errors of the solutions if it exists. The error represents the deference between constant and the final value produced from substitution.

Table (4): Lindo Case 1 verification

Equation	Constant	Substitution Value	The error
Equation 1	7	7	0
Equation 2	−28	−28	0
Equation 3	49	49	0
Equation 4	1	1	0
Equation 5	−36	−36	0
Equation 6	32	32	0
Equation 7	18	18	0
Equation 8	=9	=9	0
Equation 9	−2	−2	0
Equation 10	21	21	0

4. Solution of Case Study using Maxima:

Enter the system in Maxima as in Figure (5)

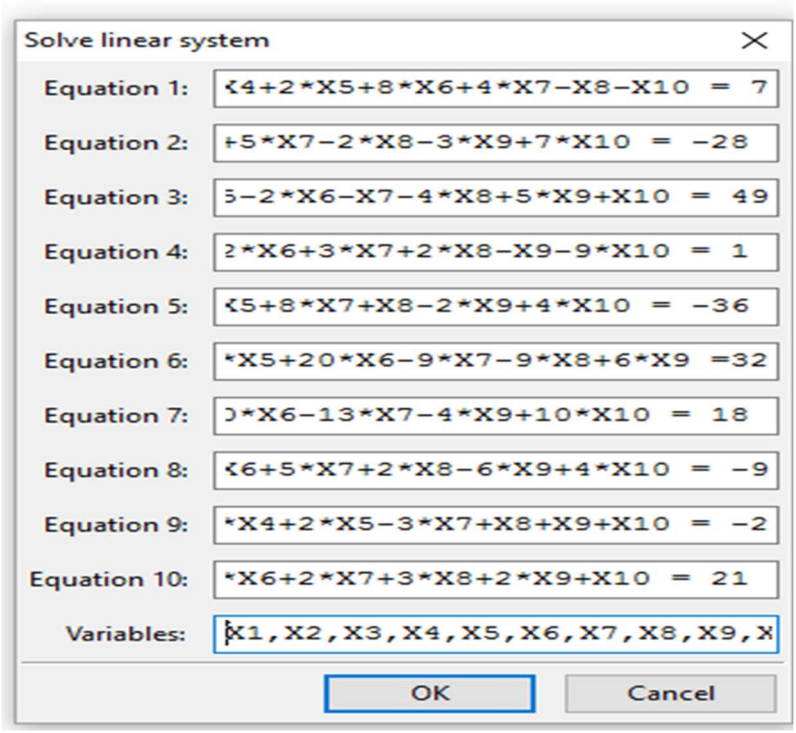


Figure (5): Case 1 Maxima input

Then the solution is in figure (6)

```
(%i2)   %,numer;
(%o2)   [X1=-2,X2=1,X3=5,X4=1.5,X5=3,X6=0.25,X7=-1,X8=6,X9=10,X10=2]
```

Figure (6): Case 1 Maxima Answer

The following Table (5) shows the answers by Maxima:

Table (5): Maxima Case 1 Answers

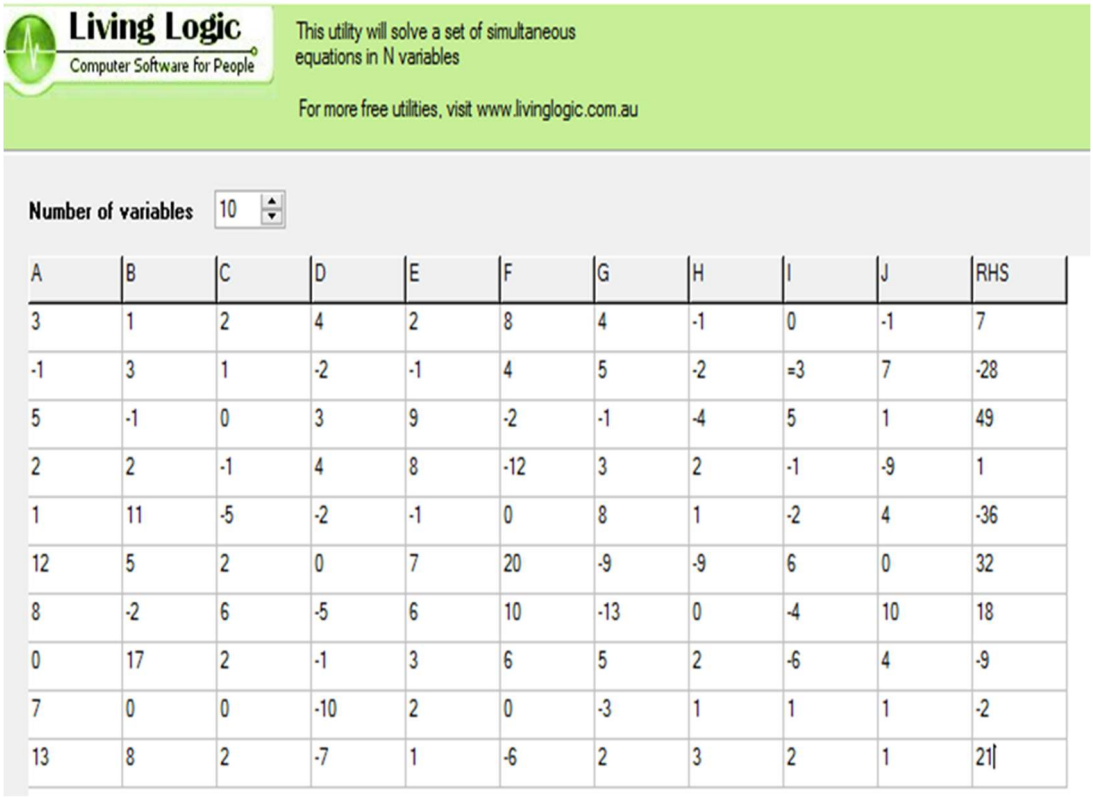
Variable	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
Value	−2	1	5	1.5	3	0.25	−1	6	10	2

Table (6) shows the substitutions of the above values of the variables in the first case study to find the errors of the solutions if it exists. The error represents the deference between constant and the final value produced from substitution.

Table (6): Maxima Case 1 verification			
Equation	**Constant**	**Substitution Value**	**The error**
Equation 1	7	7	0
Equation 2	−28	−28	0
Equation 3	49	49	0
Equation 4	1	1	0
Equation 5	−36	−36	0
Equation 6	32	32	0
Equation 7	18	18	0
Equation 8	=9	=9	0
Equation 9	−2	−2	0
Equation 10	21	21	0

5. Solution of Case Study by using SimSolve:

Enter the matrix coefficients and the constants in SimSolve, see Figure (7)



A	B	C	D	E	F	G	H	I	J	RHS
3	1	2	4	2	8	4	-1	0	-1	7
-1	3	1	-2	-1	4	5	-2	=3	7	-28
5	-1	0	3	9	-2	-1	-4	5	1	49
2	2	-1	4	8	-12	3	2	-1	-9	1
1	11	-5	-2	-1	0	8	1	-2	4	-36
12	5	2	0	7	20	-9	-9	6	0	32
8	-2	6	-5	6	10	-13	0	-4	10	18
0	17	2	-1	3	6	5	2	-6	4	-9
7	0	0	-10	2	0	-3	1	1	1	-2
13	8	2	-7	1	-6	2	3	2	1	21

Figure (7): SimSolve input

Then the solution as in Figure (8)

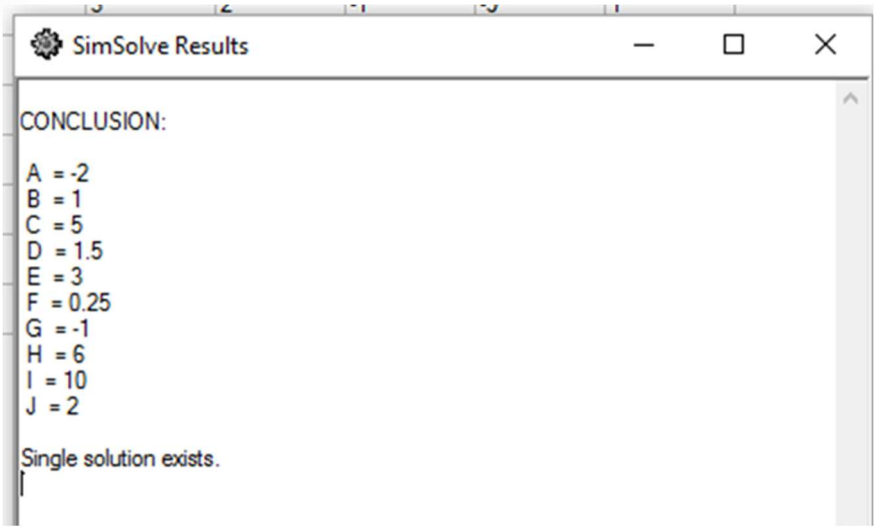


Figure (8): SimSolve Output

The following Table (7) shows the answers by SimSolve:

Table (7): SimSolve Case 1 Answers										
Variable	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
Value	−2	1	5	1.5	3	0.25	−1	6	10	2

Table (8) shows the substitutions of the above values of the variables in the first case study to find the errors of the solutions if it exists. The error represents the deference between constant and the final value produced from substitution.

Table (8): SimSolve Case 1 verification			
Equation	Constant	Substitution Value	The error
Equation 1	7	7	0
Equation 2	−28	−28	0
Equation 3	49	49	0
Equation 4	1	1	0
Equation 5	−36	−36	0
Equation 6	32	32	0
Equation 7	18	18	0
Equation 8	=9	=9	0
Equation 9	−2	−2	0
Equation 10	21	21	0

6. Conclusion

In the case study which is represents a sample of:

- High order system (10 equations).
- Number of equations = number of variables.
- The coeficients are integers.
- The system has a simultaneous soluntion.

Did not record any differences in the solutions and the results match, and no mistakes in all software solutions after verification.

As for the time and effort solves this system no later than the time and effort admission process.

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